Pricing Repo: a Model of Haircuts and Rates

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Abstract

We study the effect of the quality of collateral on repo rates and haircuts. We build a model of repo based on differences in beliefs, where Value-at-Risk and Expected Shortfall arise endogenously as sufficient statistics of the quality of collateral, i.e. its return distribution. Although a higher Expected Shortfall increases both haircuts and repo rates, a higher Value-at-Risk leads to a larger haircut and a lower repo rate. We show that while riskier borrowers face higher haircuts, they do not necessarily pay higher rates. More profitable borrowers borrow more against the same collateral at a cost of paying higher rates.

JEL Classification: D83, G23, G32

Keywords: repo, collateral, haircut, repo rate, Value-at-Risk, Expected Shortfall

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1 Introduction

Repo, a form of lending collateralized by a portfolio of securities, is a major source of short-term funding for financial institutions worldwide. The daily turnover of the global repo market is around EUR 3 trillion, with the size reaching EUR 15 trillion outstanding (International Capital Markets Association (2019)). The repo market is also systemically important: It may have played or played an important role in the propagation of the 2008 financial crisis (Gorton and Metrick (2012)).[1] Although repo contracts are widely acknowledged as key to the efficient working of almost all financial markets (International Capital Markets Association (2019), CGFS (2017)), little is known about how the equilibrium in the repo market is determined. Unlike unsecured debt, a repo has not only a price condition (interest rate), but also a non-price condition that measures the degree of collateralization (haircut), that is, the amount by which the value of the collateral exceeds the loan size. The lack of theoretical guidance for the setting of repo parameters leads to a large variety of methodologies being implemented by market participants, which "range from the intuitive to adaptions of market risk measurement techniques" (Comotto (2012b)). Empirical research shows that repo contracts are affected by both the quality of the collateral and the borrower's identity.[2] Two important questions follow. First, which measures of collateral quality are relevant to repo and how do they influence the equilibrium repo parameters? Second, how do the borrower's characteristics such as credit risk and profitability influence repo parameters?

We build a partial equilibrium model of repo based on difference in beliefs, where Value-at-Risk (VaR) and Expected Shortfall (ES) arise endogenously as sufficient statistics of the collateral's quality, that is, of its return distribution. We find that these two seemingly interchangeable metrics represent risks of different natures and, therefore, affect repo parameters

[1]See also Krishnamurthy et al. (2014), Copeland et al. (2014). See (FT, April 19, 2012 [link]) and Comotto (2012a) for an example of a broader discussion between policymakers and financial practitioners.

[2]Copeland et al. (2014) and Auh and Landoni (2016), among others, give evidence on the effect of borrower's credit risk and collateral quality on repo parameters.

differently: While a higher ES increases both haircuts and repo rates, a higher VaR leads to a larger haircut and a lower repo rate. We also show that although riskier borrowers face higher haircuts, they do not necessarily pay higher rates. At the same time, more profitable borrowers choose to borrow more against the same collateral at a cost of paying higher rates.

In our model, a penniless risk-neutral borrower has a binomial project with constant return to scale as well as an asset in place, both of which will generate a random return. The borrower believes that his project has a positive NPV. He seeks financing from competitive risk-neutral lenders. These, however, hold a different belief about his project: They believe its NPV to be negative due to a low probability of success. Given their relative pessimism, the lenders require an interest rate that the borrower deems unduly high. However, the borrower and the lenders agree on the asset return's distribution, and thus pledging the asset as collateral reduces the interest rate toward a level that the borrower deems fair.

The borrower prefers pledging the collateral to selling it because the spot market for the collateral is sufficiently illiquid: If selling the collateral, the borrower incurs liquidity costs associated with the market price impact. By contrast, repo deals happen in a different market and do not affect the spot price of the asset.[3] Thus, we implicitly assume that the lender has an advantage in liquidating the financial asset: While the borrower's liquidation horizon is shortened by the investment period of his project, the lender can sell the security over a longer time span, and therefore does not face liquidity costs.

Competitive lenders offer the borrower a repo contract, that is, a combination of a repo rate and a haircut, maximizing the borrower's surplus subject to lenders breaking even. The borrower prefers both a lower repo rate, since it increases his profit, and a lower haircut, since it allows him to borrow more and thus to scale up the project. The equilibrium is determined by two conditions. The first is the lender's break-even condition. The other is the "tangency" condition: The marginal rates of substitution of the borrower and the

[3] Although repo can be described as a sale and a repurchase of a security, the difference is that the front leg (the sale) and the back leg (the long forward) of the trade are linked as one trade (Bottazzi et al. (2012)). Unlike synthetic repo, which involves two separate deals in the spot and forward markets, the true repo does not involve a spot sale of the security and therefore does not affect the spot price.

lender with respect to the haircut and the repo rate must be equalized in equilibrium. The latter condition pins down the borrower's default probability; that is, the probability does not depend on the collateral return distribution.[4] Intuitively, the lender holds a short put option on the collateral, while the borrower holds a long call option with the same strike. When the borrower's project fails and the collateral is insufficient to cover the lender's loss, the lender's option is exercised. The tangency condition equalizes the marginal benefits of both agents from moving the strike of the options. For each agent those benefits depend on the probability that the option is exercised, which is the borrower's default probability.

We derive three main results. First, we find that the effect of the collateral's return distribution on the haircut and the repo rate can be summarized by two statistics: Value-at-Risk, that is, the maximal loss that can happen with a given probability, and Expected Shortfall, that is, the expectation of the loss conditional on the loss exceeding the VaR. Mathematically, VaR is a quantile of the loss distribution, and ES is the expectation within this quantile. Both VaR and ES are widely used in the industry with ES being often viewed as an analog or as a potential replacement for VaR. For example, Basel III proposes a shift from Value-at-Risk to Expected Shortfall for the calculation of the required capital, pointing out the inability of VaR to adequately capture "tail risk" (BIS (2016)). Instead, our model emphasizes the different nature of the two risk-metrics and suggests that they should be used together.

Second, we determine the effect of VaR and ES on equilibrium repo parameters. This comparative statics exercise does not rely on any specific form of the collateral returns distribution. We find that a larger ES leads to a larger haircut and a higher repo rate. On the other hand, a higher VaR implies a larger haircut but a lower repo rate. The latter result reflects the intrinsic difference between VaR and ES. The lender's expected loss can be represented as a product of the probability of default (PD) and the loss given default (LGD). Due to the "tangency" condition, in equilibrium PD is invariant to VaR and ES.

[4] A similar equilibrium condition arises in Adrian and Shin (2013).

Thus, when VaR increases, the lender requires more collateral to maintain the same default probability, and the haircut increases. With more collateral, the loss given default decreases. Since the probability of default is the same, but the loss given default is lower, the repo deal is less risky and the lender decreases the repo rate.

Third, we show that a borrower whom the lender deems riskier obtains less funding for the same collateral amount but may be required to pay either a higher or a lower repo rate.[5] The intuition is as follows. When facing a riskier borrower, the lender pays more attention to the collateral since the default probability is higher. Thus, she requires a higher interest rate for each haircut that satisfies her zero-profit condition. This shift is not uniform: The contracts with lower haircuts experience a larger increase in rates, since when the haircut is low, the lender is more exposed to the borrower's insolvency. As a result, a riskier borrower has a higher incentive to pledge more collateral: The gain from the repo rate reduction is higher for him than for a borrower who is less risky. However, since the loss given default is generally non-monotonic in the borrower's default probability, a riskier borrower can get either a higher or a lower repo rate, depending on the curvature of the lender's break-even condition.

The paper proceeds as follows. Section 2 provides an overview of the existing theoretical and empirical literature; section 3 sets up the model; section 4 provides the analysis of the model and presents the results; and section 5 concludes.

2 Related Literature

Repo lending is a diverse phenomenon. Although deals of different economic nature can be structured as repo, they have to be modeled differently. Hedge funds often use repo to finance the purchase of the underlying security, which is typically perceived as a consequence of the difference in beliefs about its return (Geanakoplos (2003), Simsek (2013), Gottardi

[5] While the borrower and the lender disagree about the borrower's default probability, we focus on the probability as perceived by the lender.

et al. (2019),Infante (2019)). Similarly, security lending deals also require a difference in beliefs about the security's payoffs. By contrast, we approach repo as a type of collateralized lending, where the counterparties' beliefs about the collateral are symmetric, but the lender has a lower valuation of the borrower's project (Chan and Kanatas (1985)). Unlike Chan and Kanatas (1985), we model collateral as a risky asset with a general distribution of return, which helps us find which aspects of the distribution are deemed important by counterparties in equilibrium. This setting best captures the nature of the interbank repo market where participants use repo for liquidity management and to finance positions in other instruments. However, our approach is quite general and can be applied to any case of repo (and collateralized lending) when collateral is not the source of the disagreement.

The primary purpose of this work is to find which properties of the collateral affect repo parameters, and in what way. Very few articles address this question from a normative perspective; instead most academic works refer to the methods that currently prevail in the market. Brunnermeier and Pedersen (2008) cite the commonly accepted market practice to use VaR to calculate margins. Using the distributions family given by the Extreme Value theory, Adrian and Shin (2013) justify VaR as a method for setting haircuts. By contrast, our model demonstrates the effect of collateral VaR and ES on both haircuts and rates. In addition, it does not require assumptions about any specific family of return distributions. Our result partially resembles the finding of Dang et al. (2013), who explain haircuts using Information Acquisition Sensitivity (IAS). If decomposed, IAS can be expressed in terms of VaR and ES, similar to what we find in this paper. However, Dang et al. (2013) do not determine rates in equilibrium, focusing only on haircuts. By contrast, in this paper haircuts and rates are determined jointly, which allows demonstrating that different collateral characteristics have different effects on the repo rate. Another important difference is the pure security-specific nature of the IAS, while in our model the equilibrium order of VaR and ES changes with the borrower's characteristics.[6]

[6] An obvious candidate in the list of repo determinants is collateral liquidity (see, for example Zhang (2019)). We do not speak explicitly about this component of asset quality. On the other hand, one can think

This paper is related to the broader literature on collateralized debt.[7] In most of the models, collateral is used either because of moral hazard (Stiglitz and Weiss (1981), Boot et al. (1991), Geanakoplos (2003), Fostel and Geanakoplos (2008), Geanakoplos (2010), Fostel and Geanakoplos (2011), Simsek (2013), Kuong (2015), and Gottardi et al. (2019)) or because of adverse selection (Stiglitz and Weiss (1981), Bester (1985), Chan and Kanatas (1985), Besanko and Thakor (1987a), Besanko and Thakor (1987b), Boot et al. (1991), and Infante (2013)).[8] By contrast, in our model the borrower and the lender disagree about the quality of the borrower's project: the lender is relatively more pessimistic. As a result, she asks for a higher interest rate on an uncollateralized loan than the borrower is willing to pay. If the lender refuses to lend at an affordable rate without collateral, she may be willing to decrease the interest rate once the collateral is pledged. This role of collateral is also assumed in Dang et al. (2013) and Eren (2015) and is considered in the industry to be one of the main purposes of repo.[9] Unlike the model presented here, Dang et al. (2013) and Eren (2015) explain the use of collateral by the presence of sequential deals (repo chains).

Alternative to pledging the financial asset as collateral, the borrower can sell it. In order to study collateralized debt, most theoretical works restrict the selling of the pledgeable asset. A typical way to proceed is to assume that the borrower's valuation of the collateral asset is higher (Barro (1976), Besanko and Thakor (1987a), Boot et al. (1991)), or that the collateral cannot be liquidated (Martin et al. (2014)). By contrast, Kuong (2015) and Parlatore (2019) assume that pledging the collateral is preferable to selling it due to the illiquidity in the collateral market. We follow the latter approach, assuming that if the borrower decides to sell the collateral in the spot market, his liquidation horizon will be

of the collateral price distribution as being the distribution of the *liquidation* price of the collateral. In this way liquidity component can be incorporated into the price distribution.

[7]For a more general and thorough review of previous works on collateralized debt, see Coco (2000).

[8]Empirically, the evidence is quite mixed (see Berger and Udell (1990), Gonas et al. (2004), Berger et al. (2011), and Berger et al. (2016)). Berger et al. (2016) explain the diversity of results by different nature of collateral, acknowledging that in most cases collateral is associated with more risky counterparties. This conclusion is usually seen as supporting moral hazard theories.

[9]"Long-holders can also gain incremental returns by engaging in repo transactions with cash investors for securities they own but have no immediate need to sell" (https://www.sifma.org/resources/research/us-repo-market-fact-sheet-2017). See also International Capital Markets Association (2019).

restricted by the investment period of his project. When selling the collateral quickly, the borrower will incur some liquidity costs associated with the market price impact. On the other hand, repo deals happen in a different market and do not affect the spot price of the asset. By contrast, the lender does not face liquidation costs, which reflects her flexibility in adjusting the liquidation horizon.

Most of the existing theoretical literature tries to understand the equilibrium effect of the borrower's credit risk. Bester (1985), Chan and Kanatas (1985), Besanko and Thakor (1987a), and Infante (2013) show that in adverse selection models creditworthy borrowers signal their quality by pledging more collateral, thus getting a lower rate. The opposite result (lower haircuts) is obtained by Besanko and Thakor (1987b), Boot et al. (1991), and Inderst and Mueller (2007). The model developed here predicts that borrowers with higher observable credit risk pledge more collateral for the same amount borrowed, which is in line with the empirical data on collateralized debt.[10]

Although the number of empirical studies of collateralized debt is significant, the evidence about the repo market is scarce. Auh and Landoni (2016) use a unique dataset of trades made in 2004–2007 by a hedge fund family that was attracting liquidity by means of repo contracts. They find, controlling for the collateral and for the borrower-lender pair, that the haircut and the repo rate display a negative relation. A similar fact is documented by Baklanova et al. (2019) on a dataset of bilateral repo deals. This is in line with our model, as well as with existing adverse selection theories. Chan and Kanatas (1985), Besanko and Thakor (1987b), and Besanko and Thakor (1987a) predict that haircuts and rates shift in the opposite direction as the borrower's credit risk changes. However, Auh and Landoni (2016) show interchangeability of haircuts and repo rates on a sample of short-term deals rolled over by the same counterparties. While this finding is much harder to explain with adverse selection, our paper suggests that the borrower's need for funding, captured by the return on his own project, may be the source of the variation observed in the empirical literature.

[10]See Berger et al. (2016) for a discussion of different evidence. See Julliard et al. (2019) for a recent analysis of haircut determinants in the British repo market.

Using a dataset from the US tri-party repo market, Copeland et al. (2014) find that both the borrower and the collateral fixed effects matter for determining the haircut. Still the sample studied by Copeland et al. (2014) does not allow relating repo deal parameters to the collateral return distribution.[11] Julliard et al. (2019) demonstrate that higher collateral VaR as well as lower collateral credit rating increase the haircut, similar to the effect of a higher borrower's credit risk. Baklanova et al. (2019) show that a higher absolute value of 5% VaR of the collateral leads to a higher haircut for the bilateral deals backed by US treasuries. To the best of our knowledge, no empirical paper tests the effect of alternative collateral risk metrics like ES.

In a related paper, Benmelech and Bergman (2009) show that deals backed by safer collateral have lower interest rates and higher loan-to-value ratios (lower haircuts). A similar finding is obtained by Auh and Landoni (2016) for repo collateralized by tranches of different seniority. As the authors of the latter work point out, "There is no obvious theoretical reason why the lender should take more risk (as evidenced by a higher spread) when the collateral quality drops)." We try to resolve this paradox by showing that if two portfolios differ in the tail risk of the collateral (expressed in terms of ES), then a repo deal backed by the portfolio with a higher ES receives a higher rate and a higher haircut, in line with Auh and Landoni (2016).

Finally, this work is related to the literature on risk measures. Although VaR is a widespread risk metric, in an influential paper, Artzner et al. (1999) accuse it of being incoherent, raising concerns about its practical applicability. Due to its non-sub-additivity, VaR of a portfolio can be higher than the sum of VaR of its components. This is inconsistent with a basic requirement for risk measures, questioning the applicability of VaR, at least in some cases.[12] On the other hand, ES does not have this problem (Acerbi and Tasche (2002)).

[11] A significant part of the tri-party repo market is GC (general collateral) repo, which specifies not a particular security to be pledged but rather a set of similar securities. See Copeland et al. (2014) or International Capital Markets Association (2019) for more details.

[12] A failure of sub-additivity means that a metric may assign a higher risk level to a diversified portfolio than to a concentrated one, which is an undesirable feature for a risk measure. One would assume a diversified portfolio to be *at most* as risky as its components when considered separately.

We add to the ongoing debate between the proponents of the two risk measures, showing that instead of substitutes, VaR and ES may be treated as complements and used together.[13]

3 Model

The model has two periods: $t \in \{0, 1\}$. At $t = 0$, a risk-neutral borrower has a scalable binomial investment project with constant return to scale. For each unit invested, the project will pay a cash flow $(1 + \rho)$ at $t = 1$ if successful and zero otherwise.[14] The borrower believes that the project succeeds with probability $(1 - P_B)$ and has a positive NPV: $NPV_B \triangleq (1 + \rho) \times (1 - P_B) - (1 + r_f) > 0$, where r_f is the risk-free rate. At $t = 0$, the borrower is penniless but endowed with one unit of a perfectly divisible asset (i.e., a stock) with a market price of 1. At $t = 1$, the stock pays off R, which is distributed with a CDF $F(R)$ on a bounded support $[a, b]$ $(a > 0)$, independently of the project's return. We assume that $F(R)$ is invertible; that is, $F^{-1}(x)$ exists for all $x \in [0, 1]$. We also assume that to borrower does not sell the stock due to the liquidity costs of an immediate sale in the spot market at $t = 0$.[15]

At $t = 0$, the borrower may enter a repo contract with one of competitive risk-neutral lenders. These agents are deep-pocketed, funding at a rate r_f. The contract specifies the amount borrowed (M), the amount of collateral $(N \in [0, 1])$, and the interest rate, or repo rate, r, that the borrower will need to repay at $t = 1$. At $t = 1$, if the project pays off, the borrower pays the lender the promised amount $(1 + r)M$. Otherwise, the borrower is insolvent and the lender can keep the collateral, recovering $min((1 + r)M, R \times N)$, while the

[13] In the comparative statics section we compare VaR of two portfolios that have the same value of the ES. In general, we advocate for a joint use of the two metrics.

[14] Zero payoff when the project fails helps to capture the idea that the resolution of a failing bank is typically a highly complex and time-consuming process. This is similar to Infante (2019), where lenders regard the unsecuritized liabilities of a failed borrower as worthless.

[15] To invest in the project, the borrower needs to obtain cash either by selling the stock or by borrowing. We assume that if the borrower decides to sell his stock holdings, he incurs a proportional cost c. There exists a threshold value $\tilde{c}$ of c such that the borrower decides to sell the stock if $c < \tilde{c}$. We provide the proof and find the value $\tilde{c}$ in Appendix A2. For the purpose of our study, we assume that the cost is high enough $(c > \tilde{c})$ and that the borrower does not sell the asset.

borrower retains $max(R \times N - (1+r)M, (1-N)R)$.[16] For simplicity lenders are assumed riskless.[17]

When negotiating a repo deal, counterparties usually do not discuss the value of the collateral but rather the degree of overcollateralization of the deal, commonly referred to as the haircut or the margin.

Definition 1. *The haircut h of the repo deal is the overcollateralization of the contract $(N - M)$ normalized by the amount of money borrowed: $h \triangleq (N - M)/M$.*[18]

The lowest value $h = -1$ corresponds to an uncollateralized deal, while a positive haircut means an overcollateralization. The amount borrowed is thus $M = N/(1+h)$.

We show in Appendix A1 that since $NPV_B > 0$, the borrower pledges the entire unit of the asset as collateral in equilibrium ($N = 1$).[19] The payoff of the project and the collateral return being independent, the borrower's expected utility is

$$W(r, M) = M \times (\rho - r) \times (1 - P_B) + P_B \times \mathbb{E}[max(R - (1+r)M, 0)]. \tag{1}$$

Using Definition 1, we get

$$W(r, h) = \left(\frac{1+\rho}{1+h} - \frac{1+r}{1+h}\right) \times (1 - P_B) + P_B \times \mathbb{E}\left[max\left(R - \frac{1+r}{1+h}, 0\right)\right]. \tag{2}$$

The lenders' opportunity costs of financing the borrower are $(1 + r_f)$. However, lenders hold a different belief about the project: They think that it pays back with probability

[16] Notice that the lender does not face liquidity costs when repossessing the collateral. In reality, lenders can adjust the collateral liquidation horizon, avoiding the costs associated with an immediate sale of a large portfolio.

[17] This assumption is common in the literature. Exceptions, including Dang et al. (2013) and Eren (2015), rely on the lender's default probability due to the sequential structure of the deal, when the lender in one transaction is a borrower in another one. We do not consider repo chains in this model.

[18] This definition provides a measure of collateral coverage of the contract that is closer to what is usually known as margin. Note, however, that due to the one-to-one correspondence and the closeness of the two terms we stick to Definition 1.

[19] Certainly, this statement holds only if the equilibrium with repo exists in the first place. As we show further, it depends on the lender's belief about the NPV as well.

$(1 - P_L) < (1 - P_B)$, such that the project is unprofitable; that is, $NPV_L \triangleq (1 + \rho) \times (1 - P_L) - (1 + r_f) < 0$. Therefore, the lender's utility is

$$U(r, M) = (1 - P_L) \times (1 + r)M + P_L \times \mathbb{E}[min(R, (1 + r)M)] - (1 + r_f)M. \tag{3}$$

4 Results

In this section we analyze the model and discuss the results.

4.1 Preliminaries

The following definition and notation will be convenient for what follows.

Definition 2. *The comfort return is*

$$K \triangleq \frac{1 + r}{1 + h}. \tag{4}$$

Intuitively, the comfort return is the threshold value of the collateral return, above which the haircut guarantees full coverage of the loss. For example, when $h = 0$, if the borrower is insolvent and the collateral return exceeds $K = \frac{1+r}{1+h}\big|_{h=0} = 1 + r$, the lender gets all amount due; otherwise she gets a fraction of it. On the other hand, if the haircut is positive ($h > 0$), a smaller return of collateral is sufficient to ensure the proper repayment ($K = \frac{1+r}{1+h}\big|_{h>0} < 1+r$).

Using Definition 2, one can conveniently express the utility functions (2) of the borrower and (3) of a lender in the following way:

$$W(h, K) = \left(\frac{1 + \rho}{1 + h} - K\right) \times (1 - P_B) + P_B \times \mathbb{E}\left[max\left(R - K, 0\right)\right] \tag{5}$$

and

$$U(h, K) = K + P_L \times \mathbb{E}[min(R - K, 0)] - \frac{(1 + r_f)}{(1 + h)}. \tag{6}$$

The form of equation (6) has a similar intuition to that of Merton (1974): The return on the lender's claim can be decomposed into a "riskless" (non-default) component and a short put option component. Equivalently,

$$U(h, K) = K \times \left\{ 1 - P_L \times F(K) \times \mathbb{E}\left[\frac{K - R}{K} \middle| R < K \right] \right\} - \frac{(1 + r_f)}{(1 + h)}. \tag{7}$$

This formulation allows for a straightforward interpretation. The amount K is the non-default return to the lender. The term in curly brackets can be rewritten as $\{1 - PD \times LGD\}$ from the point of view of the lender, where $PD = P_L F(K)$ is the probability of default, while $LGD = \mathbb{E}\left[\frac{K-R}{K} \middle| R < K \right]$ is the loss given default, the expected fraction that the lender fails to recover in default.

The lenders offer the borrower a repo contract, that is, a combination of a haircut and a rate. Their competitiveness implies that they undercut each other, until the break-even condition binds. Figure 1 demonstrates the lender's break-even condition, or the zero-profit indifference curve, which plays an important role in the following analysis. If the distribution of R is truncated above zero ($a > 0$), there exists a haircut h_f such that the contract is risk-free (usually referred to as the "risk-free haircut").[20] A further increase of the haircut in excess of h_f does not benefit the lender. Thus, for haircuts higher than h_f, the indifference curve is vertical. On the other hand, when no collateral is posted, that is, when $M \to \infty$ or $(1 + h) \to 0$, there always exists a rate that allows uncollateralized lending for a fixed level of the lender's utility.[21]

The curvature of the lender's zero-profit condition depends on the distribution assumptions. In terms of the truncated normal distribution used in the illustrative example, the relevant parameter is the variance. If the variance is zero (degenerate case), the indifference curve is a piece-wise straight line. As the variance grows, the line becomes smoother and the distance from the "angle" increases.

[20] If the distribution of the return R is not truncated on the left, i.e., when the price of the risky asset can drop to zero, this value does not exist ($h_f \to \infty$).

[21] Indeed, there is no credit rationing in the model.

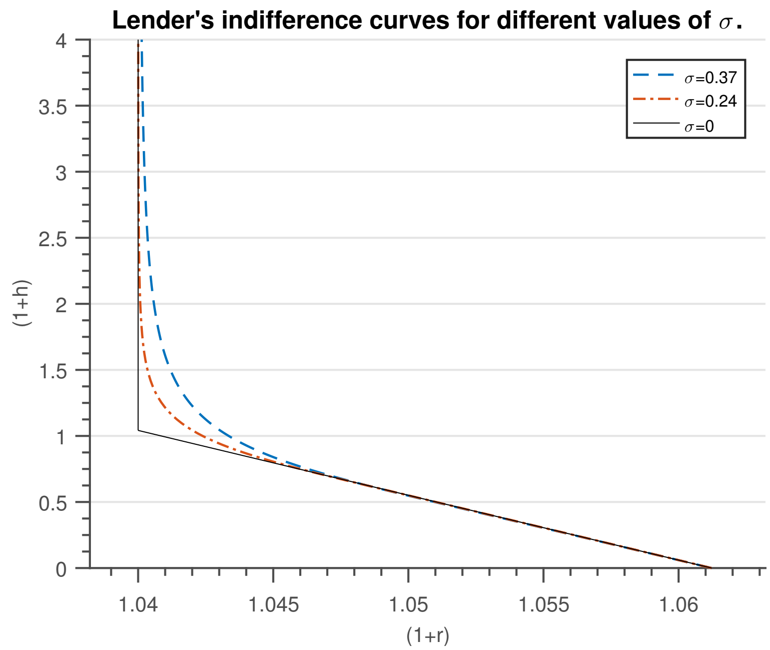

Figure 1: The lender's break-even conditions are built for R distributed according to a truncated normal distribution with different levels of variance σ. The riskless rate and the lender's belief about the probability of the project paying back are $r_f = 0.07$ and $P_L = 0.02$.

4.2 Equilibrium

In equilibrium, lenders break even; the borrower maximizes his utility subject to the lenders' break-even condition, which gives the equilibrium contract.

Definition 3. *The repo market equilibrium is a contract* (r_{eq}, h_{eq}) *such that the borrower's utility (2) is maximized subject to the lender's break-even condition, as follows:*

$$(1 + r_f) = (1 + r_{eq}) + P_L \times \mathbb{E}\left[min\left(R - K_{eq}, 0\right)\right] \times (1 + h_{eq}), \tag{8}$$

where $K_{eq} = \frac{1+r_{eq}}{1+h_{eq}}$.

Proposition 1. *The equilibrium repo rate and haircut are determined by equations (9) and (10), respectively as follows:*

$$1 + r_{eq} = (1 + r_f) \times \left(1 + P_L \times \alpha \times \left[\frac{1 - ES(\alpha)}{1 - VaR(\alpha)} - 1\right]\right)^{-1} \tag{9}$$

and

$$1 + h_{eq} = [1 - VaR(\alpha)]^{-1} \times (1 + r_f) \times \left(1 + P_L \times \alpha \times \left[\frac{1 - ES(\alpha)}{1 - VaR(\alpha)} - 1\right]\right)^{-1}, \tag{10}$$

where

$$\alpha \triangleq \frac{(1+\rho)(1-P_B) - (1+r_f)}{(1+\rho)(1-P_B)P_L - P_B(1+r_f)}, \tag{11}$$

$$VaR(\alpha) \triangleq 1 - F^{-1}(\alpha), \tag{12}$$

$$ES(\alpha) \triangleq 1 - \mathbb{E}\left(R | R < 1 - VaR(\alpha)\right). \tag{13}$$

Proof: See Appendix A3.

Equations (9) and (10) defining the equilibrium haircut and rate are of a simple form. Expression (9) can be rewritten as

$$1 + r_f = (1 + r_{eq}) \times \{1 - PD_{eq} \times LGD_{eq}\}.$$

In equilibrium the repo rate is set so that the lender's expected profit is zero.

The equilibrium haircut equation (10) can be rearranged in the following way:

$$\frac{1 + r_f}{1 + h} = (1 - VaR) \times (1 - PD) + (1 - ES) \times PD.$$

The left-hand side is the lender's financing costs, while the right-hand side is her revenues. With probability $(1 - PD)$, she expects to get the return $(1 - VaR) = K$, that is, to recover $(1 + r)M$ either directly from the borrower or due to the collateral. With probability PD, the borrower defaults, and the lender receives $\mathbb{E}[R|R < K] = (1 - ES)$. The equilibrium haircut balances the financing costs and the benefits of the lender so that she breaks even.

The equilibrium rate (9) and the haircut (10) are determined by two conditions. The first is the lenders' break-even condition (8). As we show in Appendix A4, the lender's break-even condition has a negative slope in the (rate-haircut) space. Therefore, she perceives haircuts and rates as substitutes. On the other hand, (11) can be rewritten as

$$1 + r_{eq} = (1 + h_{eq}) \times F^{-1}(\alpha). \tag{14}$$

Equation (14) follows directly from the tangency condition of the borrower's and lender's indifference curves and suggests complementarity between the haircut and the repo rate. This is not surprising since the lender benefits from higher haircuts and higher rates, while the borrower prefers lower values of both. These complementarity and substitution forces jointly determine the equilibrium.

Notice that equation (11) pins down the borrower's default probability, $PD = P_L \times \alpha$. It turns out that in equilibrium the default probability does not depend on the form of the collateral return distribution. Intuitively, the lender holds a short put option on the collateral, while the borrower holds a long call option with the same strike K. When the borrower's project fails and the collateral is insufficient to cover the lender's loss, the lender's option is exercised. The tangency condition equalizes the marginal benefits of both agents from moving the strike of the options. For each agent those benefits depend on the probability that the option is exercised, which is the borrower's default probability PD. While PD is fixed by the tangency condition (11), any change in the distribution form changes the strike K, affecting the haircut and the repo rate.

Together with (12), equation (11) gives a VaR condition similar to the one obtained by Adrian and Shin (2013). In our case, however, equation (11) is insufficient to determine both the haircut and the rate. In equilibrium the effect of ES comes through the lender's break-even condition. Another difference is that compared to the results of Adrian and Shin (2013), our model does not depend on the form of the returns' distribution.

Equation (11) also defines under what conditions the borrowing contract is collateralized. Since $F(K)$ is a probability, we require that $F(K) < 1$, which holds if

$$NPV_L \triangleq (1 + \rho) \times (1 - P_L) - (1 + r_f) < 0. \tag{15}$$

The intuition is simple. In our model the collateral is needed because of the difference in beliefs between the borrower and the lender. The necessary condition for the borrower to undertake the project is $NPV_B > 0$, but if the lender agrees with him ($NPV_L > 0$), there is no need for collateral. However, if the lender is sufficiently pessimistic about the value of the project ($NPV_L < 0$), she is unlikely to finance it unless the collateral is pledged.[22]

Proposition 1 demonstrates that the distribution of the collateral price affects the equilibrium only through VaR and ES. The model does not require any particular specification of the density function; the two metrics turn out to be sufficient statistics for any distribution law.[23] Meanwhile, according to equation (11), the probability level $\alpha \triangleq F(K)$ depends on the borrower-specific characteristics. This observation leads to the following prediction:

***Prediction* 1.** *VaR and ES are sufficient statistics of the collateral returns distribution. Jointly, they fully determine the effect of collateral on the haircut and the repo rate. The relevant level of VaR and ES depends on the parameters of the borrower (ρ, P_L and P_B) and on the risk-free rate r_f.*

The importance of VaR is empirically shown by Baklanova et al. (2019) and Julliard et al. (2019). We are unaware of any paper that tests the influence of ES on repo parameters.

The problems of the borrower and the lender have a graphical representation in the $\big((1+r),(1+h)\big)$ space. Competitive lenders undercut each other until the zero-utility indifference curve is reached. The borrower needs to pick a point on the curve that maximizes his utility. An example of equilibrium is provided in Figure 2.

[22]Since the lender is deep-pocketed and the borrower's project has constant return to scale, if $NPV_L > 0$, the lender extends an infinite loan to the borrower and both agents attain infinite profits. Thus, the borrower and the lender are indifferent to whether the collateral is pledged. We do not consider this case in the model.

[23]We still have to require the CDF of the collateral returns distribution to be invertible.

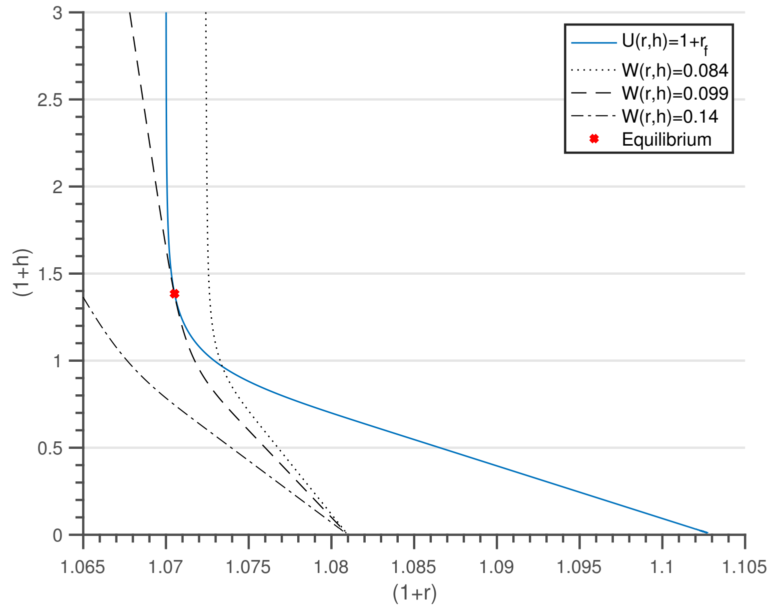

Figure 2: The equilibrium in the repo market is given by the tangent point of the lender's participation constraint ($U(r, h) = 1 + r_f$) and the borrower's indifference curve. Less steep lines correspond to a higher borrower's utility level. The borrower's and lender's parameters in this example are: $P_l = 3\%, P_b = 0.78\%, r_f = 7\%, \rho = 8.1\%$.

4.3 Comparative statics of the equilibrium

First, we consider the sensitivity of the equilibrium rate and haircut to the two main parameters of the borrower's project (the probability P_L and the return rate of the project ρ).[24] This exercise allows us to derive cross-sectional predictions about the parameters of contracts offered to different borrowers with the same collateral. Next we define the reaction of the equilibrium contract parameters to changes in the collateral return distribution. However, this task is not straightforward. What types of distribution changes should be considered? In order to answer this question, we introduce the concepts of a "quantile-preserving spread" and an "over-the-quantile spread" and limit the study to the variation of these two measures.

[24] Although the model contains two variables related to the probability of the bad state, P_B and P_L, only the lender's opinion is of interest for our analysis. One can show that an increase in the borrower's relative optimism (growth of $P_L - P_B$ keeping P_L fixed) leads to similar comparative statics as an increase in ρ.

4.3.1 The effect of the characteristics of the project

The following Corollary 1.1 summarizes the effects of borrower-specific parameters on the equilibrium contract (r_{eq}, h_{eq}).

Corollary 1.1. *The equilibrium haircut increases with a rise in P_L or with a reduction in ρ. The equilibrium repo rate r_{eq} increases in the borrower's project return ρ. The change of the equilibrium repo rate r_{eq} in response to an increase in P_L depends on the local characteristics of the returns' probability distribution.*

$$\frac{dh_{eq}}{d\rho} < 0; \qquad \frac{dr_{eq}}{d\rho} > 0; \qquad \frac{dh_{eq}}{dP_L} > 0;$$

$$\frac{dr_{eq}}{dP_L} \begin{cases} > 0 & if \quad \frac{\kappa \times (1-ES(\alpha))}{ES(\alpha)-VaR(\alpha)} < \epsilon_K^F \\ < 0 & if \quad \frac{\kappa \times (1-ES(\alpha))}{ES(\alpha)-VaR(\alpha)} > \epsilon_K^F \end{cases}, \tag{16}$$

where $\epsilon_K^F \triangleq \frac{F_K'(K_{eq})K_{eq}}{F(K_{eq})}$ is the elasticity of the CDF $F(\cdot)$ at K_{eq}, $\kappa = \left(1 - \frac{P_B}{P_L}\frac{(1+r_f)}{(1+\rho)(1-P_B)}\right)^{-1} > 1$.

Proof: See Appendix A5.

According to the Corollary 1.1, borrowers who have access to more profitable projects agree to pay a higher repo rate in order to get a lower haircut and thus to attract more funds. This happens because ρ has no direct effect on the lenders' utility, and the equilibrium point shifts along the lender's break-even condition. This result yields the following prediction.

***Prediction* 2.** *Borrowers with lower individual project returns agree to a higher haircut and a lower repo rate.*

Prediction 2 suggests another possible explanation for the interchangeability of haircuts and rates documented by Auh and Landoni (2016) and Baklanova et al. (2019), which, importantly, does not rely on adverse selection. Auh and Landoni (2016) find that deals with the same lender (e.g., rollovers of the same transaction) display strong interchangeability of haircuts and rates, which is hard to explain using existing theories. Since these are short-term deals between the same counterparties, adverse selection is unlikely to play a role in determining the parameters. Our model shows that this pattern may be explained by the borrower financing projects with different returns. In fact, one can interpret ρ more broadly as a proxy for the borrower's liquidity need. A borrower with a higher ρ is willing to pay a higher rate to obtain additional money at $t = 0$. Since borrowers' liquidity needs may change on a daily basis, they may trade off a higher amount borrowed $M = 1/(1 + h)$ for a lower price $(1 + r)$, causing the observed variation in sequential repo deals.

By contrast, with the change in ρ, changes in the probability P_L affect both equilibrium conditions. As P_L grows, since the lender views default as more likely, she tends to become more concerned with collateral risks of lower probability. As a result, she increases the repo rate for each level of haircut on the zero-profit indifference curve; the increase is especially pronounced for low haircut contracts as for the riskiest. This change affects the borrower in two ways. First, the previous utility level is unattainable. Second, the curvature of the lender's break-even condition changes, making the contracts with higher haircuts relatively more attractive than before, since for such contracts the increase in the repo rate is less pronounced. Therefore, the equilibrium haircut increases. However, the effect of a rise in P_L on the repo rate depends on the properties of the return distribution in the equilibrium point, as shown in the Corollary 1.1. The equilibrium repo rate may increase if elasticity of $F(K)$ is sufficiently high, or decrease if it is below the threshold. To summarize, Corollary 1.1 leads to the following prediction.

***Prediction* 3.** *Borrowers with a higher P_L agree to a higher haircut. The repo rate may be higher or lower dependent on the local properties of the collateral return distribution.*

Our finding supports the notion that the borrower's credit risk affects not only the repo rate but also the haircut. Prediction 3 is in line with Copeland et al. (2014) and Julliard et al. (2019), who find that borrowers with higher P_L obtain financing at a higher haircut. This contradicts a common premise that the haircut should be only collateral-specific, and the repo rate only borrower-specific. Instead, as we show, both haircut and repo rate should take into account the counterparty credit risk.

4.3.2 The effect of the probability distribution of returns

To determine the effects of changes in the collateral return distribution, we introduce the following definitions.[25]

Definition 4. *A continuous random variable Y_1 with a CDF $G_1(y)$ is an α-**quantile-preserving spread (QPS)** of a continuous random variable X with a CDF $F(x)$ if:*

1. $F^{-1}(\alpha) = G_1^{-1}(\alpha)$, *and*
2. $\mathbb{E}(X|X < F^{-1}(\alpha)) > \mathbb{E}(Y_1|Y_1 < G_1^{-1}(\alpha))$.

Definition 4 focuses on changes in the probability distribution that happen within the event $X < J$ for $J = F^{-1}(\alpha)$, leaving the probability of the event unchanged. One example of a quantile-preserving spread is an increasing probability of extreme events within the quantile. In terms of the risk measures used in the industry, quantile-preserving spreads affect the value of the ES, leaving the VaR unchanged.

Definition 5. *A continuous random variable Y_2 with a CDF $G_2(y)$ is an α-**over-the-quantile spread (OTQS)** of a continuous random variable X with a CDF $F(x)$ if:*

1) $G_2^{-1}(\alpha) > F^{-1}(\alpha)$, *and*
2) $\mathbb{E}(X|X < F^{-1}(\alpha)) = \mathbb{E}(Y_2|Y_2 < G_2^{-1}(\alpha))$.

An over-the-quantile spread is characterized by a decrease in the quantile $F^{-1}(\alpha)$, holding the conditional expectation within the event $\{X < F^{-1}(\alpha)\}$ fixed. Definition 5 describes the type of change that affects the $VaR(\alpha)$ holding the value of the $ES(\alpha)$ fixed. Examples of random variables Y_1 and Y_2 together with the initial variable X are plotted in Figure 3.

OTQS and QPS are introduced in order to demonstrate the effects of VaR and ES on the equilibrium repo contract. Proposition 1 defines VaR and ES in equations (12) and (13), as

[25] Here and later we assume that all the cumulative distribution functions are invertible and that the corresponding conditional moments exist.

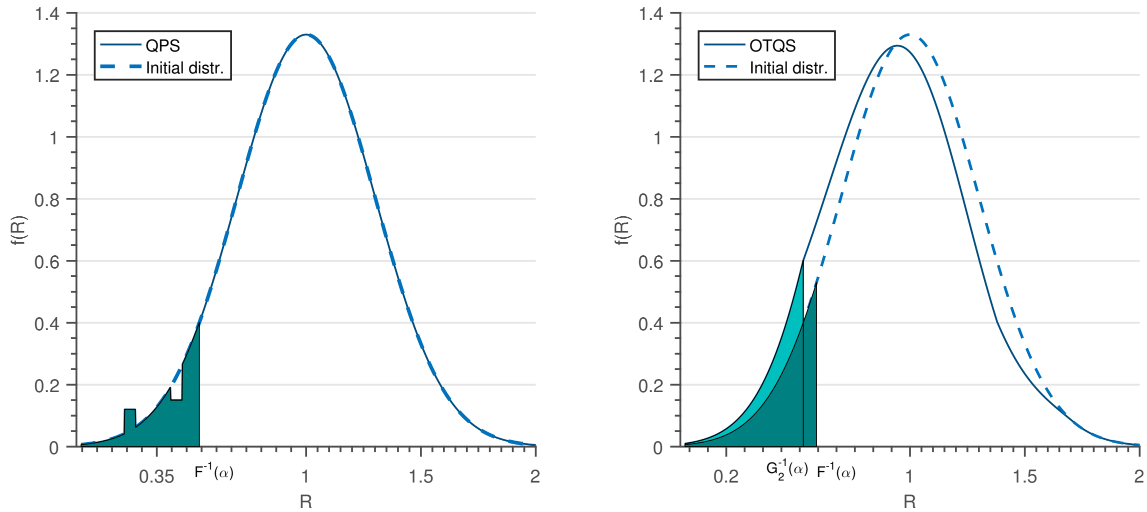

Figure 3: Examples of a quantile-preserving spread and of an over-the-quantile spread.

follows:

$$VaR(\alpha) \triangleq 1 - F^{-1}(\alpha) \qquad \text{and} \qquad ES(\alpha) \triangleq 1 - \mathbb{E}\left(X|X < 1 - VaR(\alpha)\right).$$

Since it follows from equation (11) that $K = F^{-1}(\alpha)$, where α is constant, a change to the function $F^{-1}(\cdot)$ should be considered. Apparently, $d(VaR(\alpha)) = -dF^{-1}(\alpha)$, so a shock to the VaR can be summarized by a shift of the quantile of order α. In the model $ES(\alpha) = ES(VaR(\alpha), \alpha)$, so changing the $VaR(\alpha)$ would, in general, affect the $ES(\alpha)$ as $K = F^{-1}(\alpha)$ shifts. OTQS restricts the variation of ES, allowing ES to adjust to the fluctuations of the VaR only in a particular manner, leaving the $ES(\alpha)$ unaltered. On the other hand, QPS is a change to ES that does not affect VaR. This leaves QPS and OTQS orthogonal and allows characterizing any relevant transformation of $F(\cdot)$ in terms of the two spreads. Similarly, any transformation that cannot be described in terms of QPS and OTQS does not affect the equilibrium.

Corollaries 1.2 and 1.3 point out the effects of exogenous changes to ES and VaR, respectively, on the following equilibrium parameters:

Corollary 1.2. *An (isolated) increase in the Expected shortfall* $\big(ES(\alpha)\big)$ *of the collateral* *returns distribution R affects the haircut and the repo rate in the following way:*

$$\left.\frac{dh}{dES(\alpha)}\right|_{VaR(\alpha)=const} > 0, \tag{17}$$

and

$$\left.\frac{dr}{dES(\alpha)}\right|_{VaR(\alpha)=const} > 0, \tag{18}$$

where the change to the distribution parameter $\left(dES(\alpha)|_{VaR(\alpha)=const}\right)$ *is defined in terms of an* α*-quantile-preserving spread.*

Proof: See Appendix A6.

Corollary 1.3. *An (isolated) increase in the Value-at-Risk* $\left(VaR(\alpha)\right)$ *of the collateral returns distribution R affects the haircut and the repo rate in the following way:*

$$\left.\frac{dh}{dVaR(\alpha)}\right|_{ES(\alpha)=const} > 0; \tag{19}$$

$$\left.\frac{dr}{dVaR(\alpha)}\right|_{ES(\alpha)=const} < 0, \tag{20}$$

where the change to the distribution parameter $\left(dVaR(\alpha)|_{ES(\alpha)=const}\right)$ *is defined in terms of an* α*-over-the-quantile spread.*

Proof: see Appendix A7.

Corollaries 1.2 and 1.3 illustrate the effects of the collateral-specific characteristics, yielding the following two predictions.

***Prediction* 4.** *Of two portfolios, A and B, with the same level of* $VaR(\alpha)$ *offered as collateral candidates, if portfolio A has a higher* $ES(\alpha)$*, then the contract backed by portfolio A has a higher haircut and a higher repo rate.*

***Prediction* 5.** *Of two portfolios, A and B, with the same level of* $ES(\alpha)$ *offered as collateral candidates, if portfolio A has a higher* $VaR(\alpha)$*, then the contract backed by portfolio A has a higher haircut and a lower repo rate.*

First of all, any change in the returns distribution that affects the equilibrium can be summarized in terms of the over-the-quantile and quantile-preserving spreads. The latter has a straightforward effect on the equilibrium parameters. Since a change to the expectation taken over a quantile of the distribution has no effect on (11), the comfort return stays the same. This means that the haircut and repo rate grow or decrease together, displaying the *complementarity effect*. When the security becomes riskier in terms of the quantile-preserving spread $\big(ES(\alpha)\big)$, both, the repo rate and the haircut go up, in order to offset the decrease in the loss given default.

Prediction 4 provides an explanation for another empirical result in Auh and Landoni (2016). Studying a sample of repo deals originated between the same counterparties and collateralized by different tranches of the same collateralization, they show that deals backed by senior tranches enjoy lower repo rates and lower haircuts. This finding suggests that riskier collateral induces the lender to take more risk (as captured by a higher interest rate). However, we show that if the difference between two securities pledged is described in terms of the QPS (e.g., if the tranches of lower seniority are distinguished by higher tail risk), then both the haircut and the repo rate are bigger when the collateral has a higher ES.

On the other hand, consider the change in the VaR in terms of an OTQS. This change affects both conditions — the lender's participation constraint and the equation (11) that links comfort return to VaR as shown before. These two conditions are responsible for the substitution and complementarity effects, respectively. Corollary 1.3 indicates that the effect of the lender's utility condition overweights, leading to the rate and haircut serving as substitutes. The intuition is simple. The lender's expected loss can be represented as a product of the PD and LGD. Due to the "tangency" condition, in equilibrium PD is invariant to VaR and ES. Thus, when VaR grows, the lender requires more collateral to maintain the constant default probability, and the haircut grows. When there is more collateral, the loss given default goes down. Since the probability of default is the same, but the loss given default is lower, the repo deal is less risky and the lender decreases the repo rate.

5 Concluding Remarks

This paper presents a partial equilibrium model of repo based on difference in beliefs. It provides important cross-sectional predictions regarding the repo rate and the haircut when collateral-specific or borrower-specific parameters vary.

Firstly, we consider a crucial normative question: Which parameters of the collateral should matter to the lender? We show that the effect of the collateral returns distribution on the repo rate and the haircut can be summarized in terms of VaR and ES, the two commonly used risk measures. Although these two measures are often treated as substitutes, we show that they represent two different types of risk with different effects on repo parameters. We find that although a higher ES increases both repo parameters, a higher VaR leads to a larger haircut and a lower repo rate. Thus, in general, collateral risk cannot be summarized by just one risk metric. We suggest that finding an optimal way to use VaR and ES jointly should be preferred to comparing the two in search of the single optimal risk measure.

Secondly, we study the effect of the borrower's characteristics on repo parameters. We show that riskier borrowers get higher haircuta, which is in line with empirical evidence. On the other hand, we find that riskier borrowers may obtain either higher or lower repo rates in equilibrium, depending on the local properties of the collateral returns distribution. We also find that when financing more profitable projects, borrowers choose to pay a higher rate and a lower haircut, leading to an explanation of the substitutability between the two parameters, which was identified by Auh and Landoni (2016).

Appendix

A1 - Borrower's collateral management

Below we show that when $NPV_B > 0$ and the equilibrium in the repo market exists, if the borrower funds his project by repo, he pledges his entire stock holding ($N = 1$).[26] The borrower's utility is linear in the amount of the collateral that he pledges. Therefore, it suffices to show that the utility of holding 1 unit of collateral until $t = 1$ without investing is less than the utility financing the project via repo.

In terms of per-unit returns, if the borrower keeps the collateral, he gets $\mathbb{E}[R]$. If he decides to pledge the asset as the collateral and to invest (assuming that the equilibrium in the repo market exists), he gets $(1-P_B)\frac{\rho-r}{1+h}+(1-P_B)\mathbb{E}[R]+P_B\mathbb{E}[max(R-K,0)]$, in terms of returns. The component $\mathbb{E}[max(R-K,0)]$ can be expressed from (6). The difference between the utility of repo financing and the utility of keeping the collateral is

$$(1-P_B)\frac{\rho-r}{1+h}+(1-P_B)\mathbb{E}[R]+P_B\mathbb{E}[max(R-K,0)]-\mathbb{E}[R]$$

$$=\frac{NPV_B}{(1+h)}+\left(1-\frac{P_B}{P_L}\right)\frac{(r_f-r)}{(1+h)}.$$

The first component is the NPV of the borrower, scaled by the amount of money that he is able to attract. The smaller the haircut, the more investment he attracts per unit of collateral pledged. The second component is negative and depends on the measure of disagreement $\frac{P_B}{P_L}$ and on the repo rate that will be paid by the borrower in equilibrium. If the borrower and the lender agree upon the probability of the bad state, this term disappears. Then, the borrower invests in the project whenever $NPV_B > 0$, since without disagreement, due to the Bertrand competition among the lenders, he is able to gain the entire Net Present Value of the project. On the other hand, in the extreme case when $P_B = 0$, the expression

[26] The proof, although simple, relies on the properties of the equilibrium with repo and on definitions that are not introduced at the moment of reference in section 3, Definitions 2 and 3, and Proposition 1.

reduces to $\frac{\rho-r}{(1+h)}$. This means that the borrower uses all the collateral to invest whenever the equilibrium repo rate is below the return rate on his project.

Rewriting the expression, we obtain

$$\frac{NPV_B}{(1+h)} + \left(1 - \frac{P_B}{P_L}\right)\frac{(r_f - r)}{(1+h)} = -K\left(1 - \frac{P_B}{P_L}\right) + \frac{(1+\rho)(1-P_B) - \frac{P_B}{P_L}(1+r_f)}{(1+h)}.$$

To sign the resulting expression, one needs to use the equilibrium value of the haircut from (10) rewritten in terms of VaR and ES. Denote $A \triangleq 1 - VaR(\alpha)$ and $B \triangleq 1 - ES(\alpha)$. Notice that A, $B > 0$. Therefore, (10) may be rewritten as

$$\frac{1}{1+h} = \frac{A(1-\alpha P_L) + \alpha P_L B}{1+r_f}.$$

Then

$$-K\left(1 - \frac{P_B}{P_L}\right) + \frac{(1+\rho)(1-P_B) - \frac{P_B}{P_L}(1+r_f)}{(1+h)}$$

$$= \frac{1}{(1+r_f)}\left\{(1+\rho)(1-P_B)[A(1-\alpha P_L) + B\alpha P_L] - (1+r_f)[A(1-\alpha P_B) + B\alpha P_B]\right\}$$

$$= \frac{1}{(1+r_f)}\left\{A[NPV_B - \alpha((1+\rho)(1-P_B)P_L - (1+r_f)P_B)] + B\alpha[(1+\rho)(1-P_B)P_L - (1+r_f)P_B]\right\}$$

$$= \frac{1}{(1+r_f)}\left\{B \times NPV_B\right\} > 0.$$

The last equality relies on equation (11).

Thus, as long as $NPV_B > 0$ and the equilibrium in the repo market exists, the borrower prefers to pledge all of his asset holdings as collateral.

□

A2 - Costs of selling the stock

We assume that if the borrower decides to sell the stock at $t = 0$, he will incur proportionate cost $c > \tilde{c}$. Due to these costs, the borrower avoids selling the stock and uses it as collateral instead. Below we derive the threshold value of $\tilde{c}$.[27]

If the borrower decides to sell the asset, he gets $1-c$ (the price short of the selling costs), which he can invest in his project and get $(1+\rho)(1-P_B)(1-c)$ at $t=1$. On the other hand, if he decides to keep the asset and to pledge it as collateral (assuming that the equilibrium in the repo market exists), he gets $\frac{(1-ES(\alpha))NPV_B}{(1+r_f)} + \mathbb{E}[R]$, as we show in Appendix A1. Since the stock is traded in a competitive market, populated by risk-neutral agents, its current price is the expected payoff discounted by the risk-free rate $(1+r_f) = \mathbb{E}[R]$. The borrower prefers repo financing to selling the asset if

$$\frac{(1-ES(\alpha))NPV_B}{(1+r_f)} + \mathbb{E}[R] > (1+\rho)(1-P_B) - (1+\rho)(1-P_B)c$$

or

$$\frac{(1-ES(\alpha))NPV_B}{(1+r_f)} > NPV_B - (1+\rho)(1-P_B)c.$$

Finally, the threshold value is

$$\tilde{c} = \frac{NPV_B}{(1+\rho)(1-P_B)}\left(\frac{r_f + ES(\alpha)}{(1+r_f)}\right),$$

where α is given by (11). If $c > \tilde{c}$, the borrower prefers keeping and pledging the collateral to selling it. Notice that $\tilde{c}$ can be positive or negative, depending on the sign of $(r_f + ES(\alpha))$.

A3 - Proof of Proposition 1

The borrower optimizes his utility conditional on the lender's break-even condition, which is her indifference curve for the utility level $U(h,K) = 0$. The equilibrium corresponds to the tangency of indifference curves of the borrower and the lender. Since we find that both agents have convex indifference curves, we further prove that if the tangency point exists, it

[27] The derivation, although simple, relies on the properties of the equilibrium with repo and on definitions that are not introduced at the moment of reference in section 3, Definitions 2 and 3, and Proposition 1.

is the global maximum of the borrower's problem.[28]

The lender's indifference curve is given by (6), as follows:

$$U(h, K) = K + P_L \times \mathbb{E}[min(R - K, 0)] - \frac{(1 + r_f)}{(1 + h)}.$$

Define the marginal rate of substitution for the lender as

$$MRS_L \triangleq \frac{dK}{d(1+h)} = -\frac{\partial U/\partial(1+h)}{\partial U/\partial K} = -\frac{(1+r_f)}{(1+h)^2(1 - P_L F(K))}.$$

The borrower's indifference curve, expressed from (2) as a function of K and h is

$$W(h, K) = \left(\frac{1+\rho}{1+h} - K\right) \times (1 - P_B) + P_B \times \mathbb{E}\left[max\left(R - K, 0\right)\right].$$

Define the borrower's marginal rate of substitution as

$$MRS_B \triangleq \frac{dK}{d(1+h)} = -\frac{\partial W/\partial(1+h)}{\partial W/\partial K} = -\frac{(1+\rho)(1-P_B)}{(1+h)^2(1 - P_B F(K))}.$$

The tangency condition is given by $MRS_L = MRS_B$, as follows:

$$F(K_{eq}) = \frac{(1+\rho)(1-P_B) - (1+r_f)}{(1+\rho)(1-P_B)P_L - P_B(1+r_f)},$$

which is equation (11) from the proposition. One can conveniently rewrite it as

$$F(K_{eq}) = \frac{NPV_B}{NPV_B - NPV_L(1 - P_B))}.$$

Remember that the borrower will invest only if $NPV_B > 0$. Obviously, for $F(K_{eq}) < 1$, the lender should believe that the NPV is negative ($NPV_L < 0$). The equation above and

[28] In this proof, we formulate the problem in the (comfort return – haircut) space, which makes the derivations more convenient. Although being equivalent, such a formulation is somewhat less common. Therefore, in the text, we stick to expressions in terms of the (repo rate – haircut) space.

the lender's indifference curve (8) give the equilibrium condition. Assuming invertibility of $F(K)$, and using the definition of comfort return, we get

$$1 + r_{eq} = (1 + r_f) \times \left(1 + P_L \times \mathbb{E}\left[min\left(\frac{R}{F^{-1}(\alpha)} - 1, 0\right)\right]\right)^{-1}$$

and

$$1 + h_{eq} = \left[F^{-1}(\alpha)\right]^{-1} (1 + r_f) \times \left(1 + P_L \times \mathbb{E}\left[min\left(\frac{R}{F^{-1}(\alpha)} - 1, 0\right)\right]\right)^{-1}.$$

Substitution of the VaR and ES definitions $VaR(\alpha) \triangleq 1 - F^{-1}(\alpha)$ and $ES(\alpha) \triangleq 1 - \mathbb{E}(X|X < 1 - VaR(\alpha))$, where $\alpha \triangleq \frac{(1+\rho)(1-P_B)-(1+r_f)}{(1+\rho)(1-P_B)P_L - P_B(1+r_f)}$ allows obtaining (9) and (10).

To see that the point (r_{eq}, h_{eq}) is indeed a maximum, first notice that the borrower's utility decreases in both h and K, while the lender's utility increases with higher h and K. Differentiating the slope of the indifference curve for the lender, one obtains

$$\frac{d^2K}{d(1+h)^2} = \frac{(1+r_f)^2 P_L f(K)}{(1+h)^4(1 - P_L F(K))^3} + \frac{2(1+r_f)}{(1+h)^3(1 - P_L F(K))} > 0.$$

Similarly, for the borrower,

$$\frac{d^2K}{d(1+h)^2} = \frac{(1+\rho)^2(1-P_B)^2 P_B f(K)}{(1+h)^4(1 - P_B F(K))^3} + \frac{2(1+\rho)(1-P_B)}{(1+h)^3(1 - P_B F(K))} > 0.$$

Comparison of the curvatures of the indifference curves in the tangency point (K_{eq}, h_{eq}) gives

$$\left.\frac{d^2K}{d(1+h)^2}\right|_{Lender} - \left.\frac{d^2K}{d(1+h)^2}\right|_{Borrower} = \frac{P_L - P_B}{(1 - P_B F(K))(1 - P_L F(K))} > 0.$$

It means that both curves are convex with negative slope and that the lender's indifference curve is more convex around the tangency point (K_{eq}, h_{eq}) than the borrower's curve. It follows that the lender's indifference curve lies higher in the (K, h) plane, that is, further from the origin. Thus, (K_{eq}, h_{eq}) is a local maximum of the borrower's problem. To prove that (K_{eq}, h_{eq}) is the global maximum, it suffices to show that there is no corner solution.

The domain of K is $[0, +\infty)$. Let the borrower's utility level at the point (K_{eq}, h_{eq}) be $w = W(K_{eq}, h_{eq})$. The borrower's indifference curve of level w may be expressed as

$$(1+h) = \frac{(1+\rho)(1-P_B)}{w + K - P_B\mathbb{E}[R] + P_B\mathbb{E}[min(R-K,0)]}, \tag{21}$$

and the lender's indifferent curve as

$$(1+h) = \frac{(1+r_f)}{K + P_L \times \mathbb{E}[min(R-K,0)]}. \tag{22}$$

Remember that the distribution of return $R \in [a, b]$ is bounded. Consider the case $K < a$. Equations (21) and (22) become, respectively,

$$(1+h) = \frac{(1+\rho)(1-P_B)}{w + K - P_B\mathbb{E}[R]}$$

and

$$(1+h) = \frac{(1+r_f)}{K}.$$

Taking the limit $K \to 0$, we obtain

$$\lim_{K\to 0}(1+h)\Big|_{Borrower} = \frac{(1+\rho)(1-P_B)}{w - P_B\mathbb{E}[R]} > 0,$$

while for the lender,

$$\lim_{K\to 0}(1+h)\Big|_{Lender} = \infty.$$

Thus, the lender's indifference curve is higher for $K = 0$.

Since there is no corner solution, while the interior solution is unique, (K_{eq}, h_{eq}) is the global maximum of the borrower's utility function.

$\square$

A4 - The curvature of the lender's break-even condition

Below we show that the lender's break-even condition is convex and has a negative slope in the (r, h) space.

Taking the differential of (8), with respect to r and h, we obtain

$$0 = dr + P_L \mathbb{E}\left[min\left(R - K, 0\right)\right] dh + P_L(1+h)K'_h \times (-F(K))dh + P_L(1+h)K'_r \times (-F(K))dr. \tag{23}$$

Simplification of (23) leads to

$$0 = (1 - F(K)P_L)dr + P_L(\mathbb{E}\left[min\left(R - K, 0\right)\right] + K \cdot F(K))dh$$

and

$$\frac{dr}{dh} = -\frac{P_L}{1 - F(K)P_L}\mathbb{E}[R|R < K]F(K) < 0. \tag{24}$$

The next step shows convexity.

Denote $\xi\big(K(r,h)\big) = \frac{dr}{dh}$. Then

$$\frac{d^2r}{dh^2} = \frac{d}{dh}\left[\xi(K)\right] = \frac{\partial\xi}{\partial K}\frac{dK}{dh} = \frac{\partial\xi}{\partial K}\left[\left.\frac{\partial K}{\partial r}\right|_{h=const} \times \frac{dr}{dh} + \left.\frac{dK}{dh}\right|_{r=const}\right] > 0.$$

□

A5 - Proof of Corollary 1.1

The first two effects are straightforward. The character of change in the equilibrium is determined by equation (11), for it is the only condition containing ρ. When ρ changes, to keep the equilibrium point on the same indifference curve of the lender, the haircut and repo rate move in opposite directions. Since $\frac{\partial\alpha}{\partial(1+\rho)} = \frac{(1-P_B)(1+r_f)(P_L-P_B)}{(1+\rho)(1-P_B)P_L-(1+r_f)P_B} > 0$, an increase in ρ will induce the repo rate to grow and the haircut to decrease.

To prove the statement about the effect of the probability P_L, differentiate equations (11) and (8) totally. Applying Cramer's rule, we ger

$$\begin{bmatrix} \frac{\epsilon_K^F}{1+r} \\ (1-P_L F(K)) \end{bmatrix} \frac{dr}{dP_L} + \begin{bmatrix} -\frac{\epsilon_K^F}{1+h} \\ P_L K F(K) \mathbb{E}\left[\frac{R}{K}\middle| R<K\right] \end{bmatrix} \frac{dh}{dP_L} = \begin{bmatrix} -\frac{\kappa}{P_L} \\ (1-\mathbb{E}\left[\frac{R}{K}\middle| R<K\right])F(K)(1+r) \end{bmatrix} \tag{25}$$

$$\frac{dr}{dP_L} = \frac{\Delta_1}{\Delta_2} \quad \text{and} \quad \frac{dh}{dP_L} = \frac{\Delta_3}{\Delta_2}.$$

$$\Delta_2 = \begin{vmatrix} \frac{\epsilon_K^F}{1+r} & -\frac{\epsilon_K^F}{1+h} \\ (1-P_L F(K)) & P_L K F(K) \mathbb{E}\left[\frac{R}{K}\middle| R<K\right] \end{vmatrix} > 0 \quad ,$$

$$\Delta_1 = \begin{vmatrix} -\frac{\kappa}{P_L} & -\frac{\epsilon_K^F}{1+h} \\ (1-\mathbb{E}\left[\frac{R}{K}\middle| R<K\right])F(K)(1+r) & P_L K F(K) \mathbb{E}\left[\frac{R}{K}\middle| R<K\right] \end{vmatrix}$$

$$= -\kappa K F(K) \mathbb{E}\left[\frac{R}{K}\middle| R<K\right] + \left(1-\mathbb{E}\left[\frac{R}{K}\middle| R<K\right]\right) F(K)(1+r)\frac{\epsilon_K^F}{1+h} > 0 \Leftrightarrow$$

$$-\kappa \mathbb{E}\left[\frac{R}{K}\middle| R<K\right] + \left(1-\mathbb{E}\left[\frac{R}{K}\middle| R<K\right]\right)\epsilon_K^F > 0 \quad \Leftrightarrow \quad \epsilon_K^F > \frac{\kappa \mathbb{E}\left[\frac{R}{K}\middle| R<K\right]}{(1-\mathbb{E}\left[\frac{R}{K}\middle| R<K\right])}.$$

Using the definitions of VaR and ES from Proposition 1, we get

$$\epsilon_K^F > \frac{\kappa(1-ES(\alpha))}{(ES(\alpha)-VaR(\alpha))},$$

or, expressing the elasticity as a function of only exogenous parameters,

$$\frac{f(F^{-1}(\alpha))F^{-1}(\alpha)}{\alpha} > \frac{\kappa(1-ES(\alpha))}{(ES(\alpha)-VaR(\alpha))}, \tag{26}$$

where $\kappa = \left(1 - \frac{P_B}{P_L}\frac{(1+r_f)}{(1+\rho)(1-P_B)}\right)^{-1} > 1$.

$$\Delta_3 = \begin{vmatrix} \frac{\epsilon_K^F}{1+r} & -\frac{\kappa}{P_L} \\ (1 - P_L F(K)) & (1 - \mathbb{E}\left[\frac{R}{K}\middle| R < K\right])F(K)(1+r) \end{vmatrix} > 0.$$

$\square$

This proof requires an illustration. Both the left-hand side and the right-hand side of (26) are non-trivial functions of α. Since $\alpha \in [0, 1]$, it is not obvious that inequality can take either sign within the domain of α. To see that this is indeed the case, consider two different borrowers. Assume for simplicity that they are both extremely overconfident, that is, they do not think that they can default ($P_B^i = 0$, $i = 1, 2$). The lender considers them to be equally risky, with default probability $P_L^i = 2\%$. Both borrowers possess equal holdings of the same financial asset, with the return distributed according to a truncated normal distribution with mean 1.07 and standard deviation 0.24. The risk-free rate is 7%. Suppose that the first borrower has a project that pays 7.05%, while the second borrower's project yields 8.81%. One can calculate that in equilibrium, the comfort returns for the first and the second borrowers are $K^1 = 0.0252$ and $K^2 = 0.8307$, respectively.

These different values of comfort return triggers the opposite comparative statics results. For the first borrower an increase in P_L^i is associated with an increase in the repo rate r, while for an equivalent change in the default probability of the second borrower, his repo rate goes down. The main reason for this contrast lies in the different dynamics of the LGD component $1 - \frac{1-ES(\alpha)}{1-VaR(\alpha)} = 1 - \mathbb{E}\left[\frac{R}{K}\middle| R < K\right]$. When P_L^i increases, the comfort return K^I goes down, reflecting an increase in the lender's dependence on the collateral. Consequently, the haircut goes up to balance the increase in the probability of default. On the other hand, as the collateralization grows, the LGD can either grow or decrease, leading to the change in the overall risk of the repo deal. In this numerical example, for the first borrower, the LGD increases and the repo rate grows, while for the second borrower, the LGD decreases and the repo rate goes down.

A6 - Proof of Corollary 1.2

From equation (11), given that the function $F(\cdot)$ is invertible, $K = F^{-1}(\alpha)$. The second equilibrium condition is the lender's indifference curve (8), which can be rewritten as

$$(1 + r_f) = (1 + r) + P_L(1 + h)(\mathbb{E}[R|R < K]F(K) - KF(K))$$

or

$$(1 + r_f) = (1 + r) + P_L(1 + h)(\mathbb{E}[R|R < F^{-1}(\alpha)]\alpha - F^{-1}(\alpha)\alpha).$$

Taking the total differential of $K = F^{-1}(\alpha)$ and the lender's indifference curve (8) gives us

$$\begin{aligned} &\frac{dr}{1+h} - \frac{Kdh}{1+h} = 0; \\ &dr - \frac{(r - r_f)}{1+h}dh = -\left[P_L(1 + h)\alpha\right] d\mathbb{E}[R|R < F^{-1}(\alpha)]. \end{aligned}$$

Representing the system in a convenient form, we have

$$\begin{bmatrix} \frac{1}{1+h} \\ 1 \end{bmatrix} \frac{dr}{d\mathbb{E}[R|R < F^{-1}(\alpha)]} + \begin{bmatrix} -\frac{K}{1+h} \\ -\frac{(r-r_f)}{1+h} \end{bmatrix} \frac{dh}{d\mathbb{E}[R|R < F^{-1}(\alpha)]} = \begin{bmatrix} 0 \\ -\left[P_L(1+h)\alpha\right]. \end{bmatrix}.$$

By Cramer's rule, $\frac{dr}{dd\mathbb{E}[R|R<F^{-1}(\alpha)]} = \frac{\Delta_1}{\Delta_2}$ and $\frac{dh}{d\mathbb{E}[R|R<F^{-1}(\alpha)]} = \frac{\Delta_3}{\Delta_2}$, where $\Delta_1 < 0$, $\Delta_2 > 0$ and $\Delta_3 < 0$. Thus, $\frac{dr}{d\mathbb{E}[R|R<F^{-1}(\alpha)]} = \frac{\Delta_1}{\Delta_2} < 0$ and $\frac{dh}{d\mathbb{E}[R|R<F^{-1}(\alpha)]} = \frac{\Delta_3}{\Delta_2} < 0$. But, as defined in equation (13), $dES(\alpha) = -d\mathbb{E}[R|R < F^{-1}(\alpha)]$.

□

A7 - Proof of Corollary 1.3

In this corollary we consider a change in $F^{-1}(\alpha)$. However, the shock to the distribution function should not be arbitrary: The expectation component $\mathbb{E}[R|R < F^{-1}(\alpha)]$ is held fixed, although the threshold value $F^{-1}(\alpha)$ is shifted.

Taking the full derivative, one obtains

$$\begin{bmatrix} \frac{1}{1+h} \\ 1 \end{bmatrix} \frac{dr}{dF^{-1}(\alpha)} + \begin{bmatrix} -\frac{K}{1+h} \\ -\frac{(r-r_f)}{1+h} \end{bmatrix} \frac{dh}{dF^{-1}(\alpha)} = \begin{bmatrix} 1 \\ [P_L(1+h)\alpha] \end{bmatrix}$$

By Cramer's rule, $\frac{dr}{dF^{-1}(\alpha)} = \frac{\Delta_1}{\Delta_2}$ and $\frac{dh}{dF^{-1}(\alpha)} = \frac{\Delta_3}{\Delta_2}$, where

$$\Delta_1 = \begin{vmatrix} 1 & -\frac{K}{1+h} \\ P_L(1+h)\alpha & -\frac{(r-r_f)}{1+h} \end{vmatrix} = \frac{(1+r_f)-(1+r)(1-\alpha P_L)}{1+h}$$

$$= \frac{(1+r_f)}{(1+h)}\left(1 - \frac{(1+r)(P_L-P_B)}{(1+\rho)(1-P_B)P_L - P_B(1+r_f)}\right).$$

To determine the sign of Δ_1, we use the equilibrium rate $(1+r_{eq})$ from (9), as follows:

$$(1+r_{eq}) = \frac{(1+r_f)}{1 - P_L\alpha\left[\frac{ES(\alpha)-VaR(\alpha)}{VaR(\alpha)}\right]} \quad \text{and}$$

$$(1+r_{eq}) - \frac{(1+\rho)(1-P_B)P_L - P_B(1+r_f)}{(P_L-P_B)}$$

$$= \frac{P_L((1+r_f)-(1+\rho)(1-P_B)) + P_L\alpha\left[\frac{ES(\alpha)-VaR(\alpha)}{VaR(\alpha)}\right]((1+\rho)(1-P_B)P_L - P_B(1+r_f))}{\left(1 - P_L\alpha\left[\frac{ES(\alpha)-VaR(\alpha)}{VaR(\alpha)}\right]\right)(P_L-P_B)}$$

$$= \frac{-P_L NPV_B + P_L\left[\frac{ES(\alpha)-VaR(\alpha)}{VaR(\alpha)}\right]NPV_B}{\left(1 - P_L\alpha\left[\frac{ES(\alpha)-VaR(\alpha)}{VaR(\alpha)}\right]\right)(P_L-P_B)} = \frac{-P_L NPV_B\left(1 - \left[\frac{ES(\alpha)-VaR(\alpha)}{VaR(\alpha)}\right]\right)}{\left(1 - P_L\alpha\left[\frac{ES(\alpha)-VaR(\alpha)}{VaR(\alpha)}\right]\right)(P_L-P_B)} < 0.$$

Therefore, $sgn(\Delta_1) = sgn\left(1 - \frac{(1+r)(P_L - P_B)}{(1+\rho)(1-P_B)P_L - P_B(1+r_f)}\right) \geq 0.$

$$\Delta_2 = \begin{vmatrix} \frac{1}{1+h} & -\frac{K}{1+h} \\ 1 & -\frac{(r-r_f)}{1+h} \end{vmatrix} = -\frac{1}{(1+h)^2}[(1+r) - (1+r_f) - (1+r)] > 0 \quad \text{and}$$

$$\Delta_3 = \begin{vmatrix} \frac{1}{1+h} & 1 \\ 1 & P_L(1+h)\alpha \end{vmatrix} < 0.$$

$\frac{dr}{dF^{-1}(\alpha)} = \frac{\Delta_1}{\Delta_2} > 0$ and $\frac{dh}{dF^{-1}(\alpha)} = \frac{\Delta_3}{\Delta_2} < 0$. But as defined in equation (12), $dVaR(\alpha) = -dF^{-1}(\alpha)$.

$\square$

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